Discussion of “Identification of Dynamic Games with Multiple Equilibria and Unobserved Heterogeneity with Application to Fast Food Chains In China” by Yao Luo, Ping Xiao, Ruli Xiao

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Outline of Presentation

In this discussion I will:

- Highlight two very interesting features of the paper:
    1. Dealing with label swapping across decompositions without monotonicity restrictions.
    2. Distinguishing between multiple equilibria and unobserved heterogeneity based on payoffs estimated using exclusion restrictions.
- Give some personal opinions on ways to extend the paper.

Disclaimer: I am somewhat new to the literature and do not have as much expertise as the authors do on the topic.

Idea: Inference from higher-order dependence

Let $w_t = (x_t, a_t)$ be the observable states and actions, and τ_t index multiple equilibria/unobserved heterogeneity. Then, with four periods of data and Markov + timing assumptions, we can pull out τ_{t+2} in the state transitions:

$$\Pr(w_{t+3}, w_{t+2}, w_{t+1}, w_t) = \sum_{\tau_{t+2}} \Pr(w_{t+3}|w_{t+2}, \tau_{t+2}) \Pr(w_{t+2}|w_{t+1}, \tau_{t+2}) \Pr(w_{t+1}, w_t, \tau_{t+2})$$

Which we can put into matrix form by fixing w_{t+2}, w_{t+1} into two possible values $w_{t+2} = (\bar{w}_{t+2}, \hat{w}_{t+2})$; $w_{t+1} = (\bar{w}_{t+1}, \hat{w}_{t+1})$ and taking a partition $z_t = z(w_t)$ such that the following Fs are invertible:

$$F_{z_{t+3}, \hat{w}_{t+2}, \hat{w}_{t+1}, z_t} = A_{z_{t+3}|\hat{w}_{t+2}, \tau_{t+2}} D_{\hat{w}_{t+2}|\hat{w}_{t+1}, \tau_{t+2}} B_{\hat{w}_{t+1}, z_t, \tau_{t+2}} \quad (1)$$

$$F_{z_{t+3}, \bar{w}_{t+2}, \hat{w}_{t+1}, z_t} = A_{z_{t+3}|\bar{w}_{t+2}, \tau_{t+2}} D_{\bar{w}_{t+2}|\hat{w}_{t+1}, \tau_{t+2}} B_{\hat{w}_{t+1}, z_t, \tau_{t+2}} \quad (2)$$

$$F_{z_{t+3}, \hat{w}_{t+2}, \bar{w}_{t+1}, z_t} = A_{z_{t+3}|\hat{w}_{t+2}, \tau_{t+2}} D_{\hat{w}_{t+2}|\bar{w}_{t+1}, \tau_{t+2}} B_{\bar{w}_{t+1}, z_t, \tau_{t+2}} \quad (3)$$

$$F_{z_{t+3}, \bar{w}_{t+2}, \bar{w}_{t+1}, z_t} = A_{z_{t+3}|\bar{w}_{t+2}, \tau_{t+2}} D_{\bar{w}_{t+2}|\bar{w}_{t+1}, \tau_{t+2}} B_{\bar{w}_{t+1}, z_t, \tau_{t+2}} \quad (4)$$

Then, noticing that the Bs duplicate and can be cancelled out via multiplication by inverses, we get:

$$F_{z_{t+3}, \hat{w}_{t+2}, \hat{w}_{t+1}, z_t} F^{-1}_{z_{t+3}, \bar{w}_{t+2}, \hat{w}_{t+1}, z_t} F_{z_{t+3}, \bar{w}_{t+1}, \bar{w}_{t+1}, z_t} F^{-1}_{z_{t+3}, \hat{w}_{t+2}, \bar{w}_{t+1}, z_t}$$
$$= A_{z_{t+3}|\hat{w}_{t+2}, \tau_{t+2}} D_{\hat{w}_{t+2}, \bar{w}_{t+2}, \hat{w}_{t+1}, \bar{w}_{t+1}|\tau_{t+2}} A^{-1}_{z_{t+3}|\hat{w}_{t+2}, \tau_{t+2}}$$

Interesting Feature 1: label swapping

We can use an eigen-decomposition on the LHS to identify $A_{z_{t+3}|\hat{w}_{t+2},\tau_{t+2}}$ up to a label index $I_{\hat{w}_{t+2}}$:

$$F_{z_{t+3},\hat{w}_{t+2},\hat{w}_{t+1},z_t} F^{-1}_{z_{t+3},\bar{w}_{t+2},\hat{w}_{t+1},z_t} F_{z_{t+3},\bar{w}_{t+1},\bar{w}_{t+1},z_t} F^{-1}_{z_{t+3},\hat{w}_{t+2},\bar{w}_{t+1},z_t}$$
$$= A_{z_{t+3}|\hat{w}_{t+2},\tau_{t+2}} D_{\hat{w}_{t+2},\bar{w}_{t+2},\hat{w}_{t+1},\bar{w}_{t+1}|\tau_{t+2}} A^{-1}_{z_{t+3}|\hat{w}_{t+2},\tau_{t+2}}$$

But we have to find $A_{z_{t+3}|\hat{w}_{t+2},\tau_{t+2}}$ for all $\hat{w}_{t+2}$. How do we ensure that $I_{\hat{w}_{t+2}}$ is consistent with one another?

- Answer: suppose that the eigenvalues $D_{\hat{w}_{t+2},\bar{w}_{t+2},\hat{w}_{t+1},\bar{w}_{t+1}|\tau_{t+2}}$ are distinctive. Then, matching on the eigenvalues will generate consistent label indices.
- Very cool result, especially since it can be checked in the data!

Interesting Feature 2: separate identification of ME and UH

Once the policy functions and state transitions are identified, it remains to *interpret* differences in actions across states as either coming from differences payoffs or multiple equilibria.

- But this cannot be done in the general case since in theory everything could come from payoffs.

Exclusion restriction: for observed and unobserved states $s = (x, \tau)$, let $s = \{s_i, s_{-i}\}$ be partitioned so that only s_i matter for π_i, such that

$$\pi_i(a_i, a_{-i}, s) = \pi_i(a_i, a_{-i}, s_i). \tag{5}$$

- Fairly standard, widely applicable.
- ME and UH can then be separated out from definitions: UH enters into π, whereas ME does not.

Thoughts on extensions: technical

1. When is the distinct eigenvalues assumption likely satisfied?
   - For example, distinct eigenvalues implies that $A_{z_{t+3}|\hat{w}_{t+2},\tau_{t+2}}$ is linearly independent, so no two unobserved states should lead to the same observed state. Can that be interpreted? More on this assumption may help establish applicability.
2. Could the identification of ME and UH through definitions approach apply in other models?
   - Perhaps Berry and Compiani's recent work "An Instrumental Variable Approach to Dynamic Models" which uses a GIV approach rather than the finite mixture approach.

Thoughts on extensions: EM counterfactuals

The distinction between ME and UH matters mainly for the counterfactual. So an ideal application could show the counterfactual implications.

- The EM algorithm gives probabilities for each latent state as well as the payoffs conditional on each latent state. Why not just use those for counterfactuals? Testing for ME/UH seems superfluous, except maybe to add power...
- Counterfactuals may be difficult to compute with multiple equilibria. Perhaps the method in Mar Reguant's working paper on "Bounding Outcomes in Counterfactual Analysis" might help.